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Vanishing Theorems in Hyperasymptotic Analysis

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In the former paper, we proved the following: Theorem[Commutative case] Let σ be a rational number ≥ 1 and

$$\{\mathcal{S}(R, a_h, b_h)\,(h=1,\ldots,N)\}$$

be a good open sectorial covering for σ of $\mathcal{D}(R,\infty)=\{\,z\,|\,+\infty>|z|>R\,\}$.. For $h=1,\ldots,N$, let $U_{h-1,h}(z)$ be an $m\times n$ matricial function defined in $\mathcal{S}_{h-1,h}(R)$ and, for some non-zero constant $\kappa_{h-1,h}$ with $\arg\kappa_{h-1,h}=-\frac{a_h+b_{h-1}}{2\sigma}$, $\exp(-\kappa_{h-1,h}z^{\frac{1}{\sigma}})$ is asymptotically developable to the formal power-series 0 and, for a complex number $\mu_{h-1,h}$,

$$z^{\mu_{h-1,h}}\exp(\kappa_{h-1,h}z^{\frac{1}{\sigma}})U_{h-1,h}(z)$$

is asymptotically developable to a formal power-series matrix $\sum_{s=0}^{\infty}U_s^{h-1,h}z^{-s}$ in the sector $\mathcal{S}_{h-1,h}(R)$.

Then, there exist a positive number $R''(\geq R)$, a formal power-series matrix $\widehat{V}(z)=\sum_{r=0}^{\infty}T_rz^{-r}$ and $m\times n$ matricial functions V_h defined in $\mathcal{S}_h(R'')$ $(h=1,\ldots,N)$ such that

(i) the relation

$$U_{h-1,h}(z)=-V_{h-1}(z)+V_h(z)$$

holds for $z\in\mathcal{S}_{h-1,h}(R'')=\mathcal{S}(R'',a_{h-1},b_{h-1})\cap\mathcal{S}(R'',a_h,b_h)$.

(ii) V_h is aymptotically developable to the formal power-series matrix $\widehat{V}(z)$ in $\mathcal{S}_h(R'')$, and for any sufficiently large number r,

$$\begin{aligned} T_r &= \sum_{(h-1,h)}\sum_{s=0}^{M-1}\sigma U_s^{h-1,h}(\kappa_{h-1,h})^{(s-r)\sigma+\mu_{h-1,h}}\Gamma((r-s)\sigma-\mu_{h-1,h}) \\ &+ O\{\Gamma((r-M)\sigma-\Re\mu_{h-1,h})\} \end{aligned}$$

provided $1\leq M<r$.

This theorem can be used to study the stucture of divergent pwer-series solutions to the non-homogeneous differential equations associated to linear ordinary differential equations, for example, Bessel equations, Whittaker equations, Weber equations and so on. In this talk, we will give a refined version of the above theorem. The result will be published as a joint-work with C. J. Howls, and A. B. Olde Daalhuis.

References

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